Comparison of frequentist confidence and Bayesian credible intervals for the normal mean ($\boldsymbol{\mu}$)

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We considered interval estimation of a scalar parameter of interest when we have uncertain prior information about some of the parameters of the sampling model. Farchione & Kabaila (2008) described a (1-α) frequentist confidence interval for the normal mean (μ) that utilizes the uncertain prior information that this mean is zero (μ = 0). This confidence interval reverts to the standard (1-α) confidence interval for μ when the data happen to strongly contradict the prior information that μ = 0. It is natural to ask how this confidence interval compares with a Bayesian credible interval in the same scenario. An improvement to the research of Farchione (2011) was carried out in this research, where we made the more realistic assumption that the variance (σ^2) is unknown.

We considered two prior distributions for (μ, σ^2) in the forms of $\pi(\mu, \sigma^2) = \big(\xi\delta(\mu) +$ $1-\xi\ \sigma-2$ and $\pi\mu{,}\sigma 2{=}\xi\delta\mu\sigma-1\ +\ 1-\xi\sigma-2,$ where $0{\leq}\xi{\leq}1$ and δ denotes the Dirac delta function. We found that the resulting Bayesian credible interval for μ reverts to the standard Bayesian credible interval for μ only for the marginal posterior distribution of μ for the latter prior distribution. This marginal posterior distribution of μ was then used to compare the frequentist 0.95 confidence interval for μ of Farchione & Kabaila (2008) with the corresponding Bayesian equi-tailed 0.95 credible interval for μ. The appropriate functions of the upper and lower endpoints of the Bayesian credible and the frequentist confidence intervals were used for comparisons. We found sufficient evidences to reasonably conclude that these functions depend on the data differently. Therefore, the findings suggest that this is yet another evidence where the Bayesian and frequentist inferences happen to disagree.